SUBJECT and GRADE	Mathematics Grade 11	
TERM 1	Week 3	
TOPIC	Quadratic Equations and Inequalities	
AIMS OF LESSON	• Interpret quadratic inequalities • Solve equations with two unknown variables also called simultaneous equations	
RESOURCES	***Paper based resources***	***Digital resources***
	Please go to the *Quadratic Equations and Inequalities* chapter in your textbook.	Where you see this icon in the lesson you can click on it to see a video that would assist you in understanding the content that is being discussed

LESSON 2 a – QUADRATIC INEQUALITIES

INTRODUCTION

Dear learner in the previous lesson you have learnt how to solve quadratic equations that contained fractions, square roots and squares.
We have learnt how to solve quadratic equations by using factors and the quadratic formula.
In this week's lesson we will expand on the previous week's lesson and also on gr 10 knowledge about linear inequalities, including how to represent it on a number line.
We have also learnt how to solve simultaneous equations using the elimination method and the substitution method in gr 10.
We will now build on this prior content knowledge to cover gr 11 content on **Quadratic Equations and Inequalities.**

CONCEPTS AND SKILLS

An inequality is a mathematical sentence that uses symbols such as $<; >; \leq; \geq$ to compare two quantities
The difference between an inequality and an equation lies in the comparison sign (equation =)
Inequalities are solved using methods similar to those you use to solve equations.
But the following rules are always applicable when working with inequalities:

- Change the direction of (reverse) the inequality sign whenever you multiply or divide by a negative number.
- Do not change(reverse) the direction of the inequality if you multiply or divide by a positive number

We will start this lesson with revision of grade 10 linear inequalities to prepare you for grade 11 content on Quadratic Inequalities.

EXAMPLE 1 Solve the following Linear inequalities

1. $x + 1 < 3$

$x < 3 - 1$

$x < 2$

2. $-\frac{3}{2}x + 3 \geq 1$

$-\frac{3}{2}x \geq -2$

$-3x \geq -4$

$x \leq \frac{4}{3}$

Remember to reverse the inequality sign if you divide by a negative

$-4(x - 3) \geq 2(x - 12)$

$-4 + 12 \geq 2x - 24$

$-4x - 2x \geq 24 - 12$

$-6x \geq -36$

$\therefore x \leq 6$

CAN YOU? Solve the following linear inequalities

1. $2x + 3 > 1$ **2.** $\frac{2x+4}{7} \geq \frac{3(x-3)}{3}$ 3. $4(x - 3) \geq 2(x - 10)$

Answers

1. $x > -1$ **2.** $x \leq 5$ **3.** $x \geq -4$

https://youtu.be/nMidru5YCvI

The following tables will refresh your memory on knowledge that is important as we proceed with grade 11 content

Inequality sign	words	Included/ excluded	Number line
$>$	Greater than	excluded (open)	
$\geq$	Greater than or equal to	included (closed)	
$<$	Less than	excluded (open)	
$\leq$	Less than or equal to	included (close)	

Inequality	Interval notation	Number line representation
$x > 2$	$x \in (2; \infty)$	
$x \geq 2$	$x \in [2; \infty)$	
$2 \leq x \leq 6$	$x \in [2; 6]$	
$2 < x < 6$	$x \in (2; 6)$	
$2 \leq x < 6$	$x \in [2; 6)$	
$2 < x \leq 6$	$x \in (2; 6]$	

Quadratic Inequalities

A quadratic inequality involves determining the values of x for which the graph of a parabola lies either above or below the x- axis. In the second term (on functions) you will learn how to sketch the graphs of parabola in much more detail. All we need now is a basic understanding of how to determine the x-intercepts of a parabola as well as its shape.

EXAMPLE 2

Solve for x in the following inequalities

2.1 $\boldsymbol{x^2 \leq 9}$

$x^2 - 4 \leq 0$

$(x+3)(x-3) \leq 0$

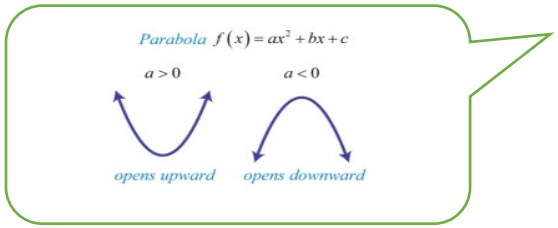
STEP 1

As in solving quadratic equations factorise to find the critical values.

STEP 2

Draw a graph to help you solve this inequality:

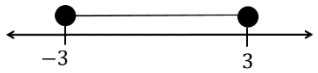
Show the x −intercepts on your graph.



The inequality sign tells us whether the answer must be read above the x-axis (where y is positive) or below the x −axis (where y-is negative)

In this case we want the values below where y is negative

The critical values are included in the final solution. Therefor we will use closed dots to draw the number line of the inequality

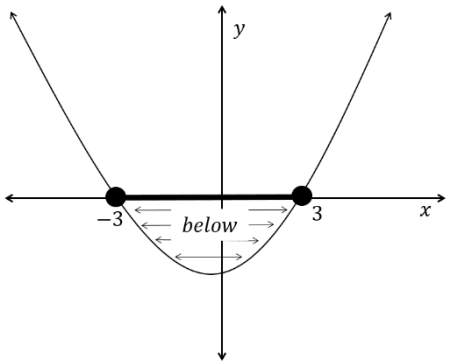


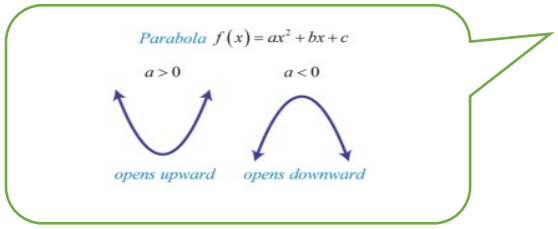
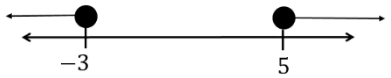
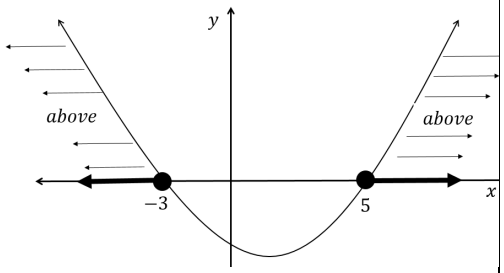
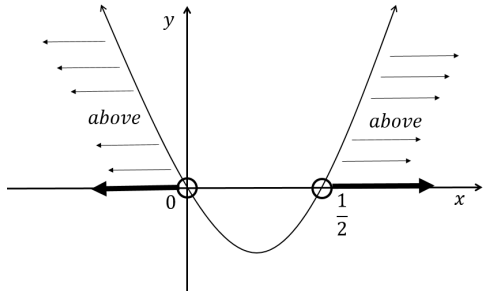
$\therefore -3 \leq x \leq 3$

STEP 3

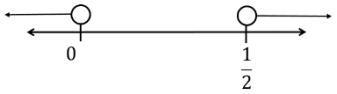
Now determine the values of x for which $x^2 - 9 \leq 0$

This is where the graph **intercepts** the $x -$ axis and where it lies **below** the x −axis.



2.2	$\boldsymbol{x^2 - 2x - 15 \geq 0}$ $(x+3)(x-5) \geq 0$ $\therefore x \leq -3 \text{ or } x \geq 5$	STEP 1 As in solving quadratic equations factorise to find the critical values. STEP 2 Draw a graph to help you solve this inequality: Show the x −intercepts on your graph.  The inequality sign tells us whether the answer must be read above the x-axis (where y is positive) or below the x −axis (where y-is negative) In this case we want the values above where y is positive The critical values are included in the final solution. Therefore, we will use closed dots to draw the number line of the inequality  STEP 3 Now determine the values of x for which $\boldsymbol{x^2 - 2x - 15 \leq 0}$ This is where the graph **intercepts** the x − axis and where it lies **above** the x −axis.	
2.3	$\boldsymbol{x < 2x^2}$ $-2x^x + x < 0$ $2x^2 - x > 0$ $x(2x-1) > 0$ $x < 0 \text{ or } x > 6$	• write in standard form • make the coefficient of x^2 positive, multiply by-1 • factorise to find the critical values. • Draw a graph The inequality sign tells us whether the answer must be read above the x-axis (where y is positive) or below the x −axis (where y-is negative) In this case we want the values above where y is positive	

- The critical values are NOT included in the final solution. Therefor we will use OPEN dots to draw the number line of the inequality



- **Read your final answer from the graph.**

CAN YOU?

Solve the following inequalities

1. $x^2 < 16$ **2.** $x^2 + 5x - 6 \geq 0$ **3.** $x^2 \leq 4x$ **4.** $-x^2 + 3x + 4 \leq 0$

Answers:

1. $-4 < x < 4$ **2.** $x \leq -6 \ or \ x \geq 1$ **3.** $0 \leq x \leq 4$ **4.** $x \leq -1 \ or \ x \geq 4$

LESSON 2b - SIMULTANEOUS EQUATIONS

https://youtu.be/sRqGfTwhGxA

INTRODUCTION:

In grade 10 you solved simultaneous linear equations to find the point of intersection of 2 straight lines. (one $x-$ value and one y-value)
Let us see if you remember how to solve linear equations simultaneously.

EXAMPLE 3

Solve the following equations simultaneously

$2y = 2x - 1 \ldots\ldots (i) \ and \ \ 2y + x - 2 = 0 \ldots\ldots (ii)$

$2y + x - 2 = 0$
$x = 2 - 2y \ldots\ldots\ldots (iii)$

$2y = 2x - 1$ …. (i)
$2y = 2(\mathbf{2 - 2y}) - 1$.
$2y = 4 - 4y - 1$
$6y = 3$
$y = \frac{1}{2}$

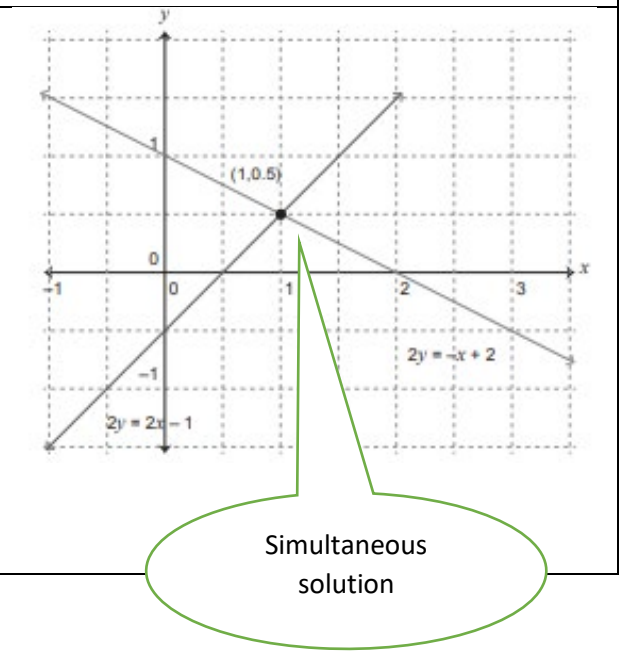
$x = 2 - 2y$
$x = 2 - 2\left(\frac{1}{2}\right)$
$x = 1$
$\therefore \boldsymbol{x = 1 \ and \ y = \frac{1}{2}} \quad \left(\mathbf{1}; \frac{1}{2}\right)$

In equation (ii) make x the subject of the formulae to get equation (iii).

substitute (iii) into (i)
You should now have an equation with only one variable

Solve for y

Substitute your answer for y into equation (i) or (ii) to solve for the second variable x



CONCEPTS AND SKILLS

We will now learn how to solve equations simultaneously when one of the equations is a quadratic equation.
When solving simultaneous equations, we can:

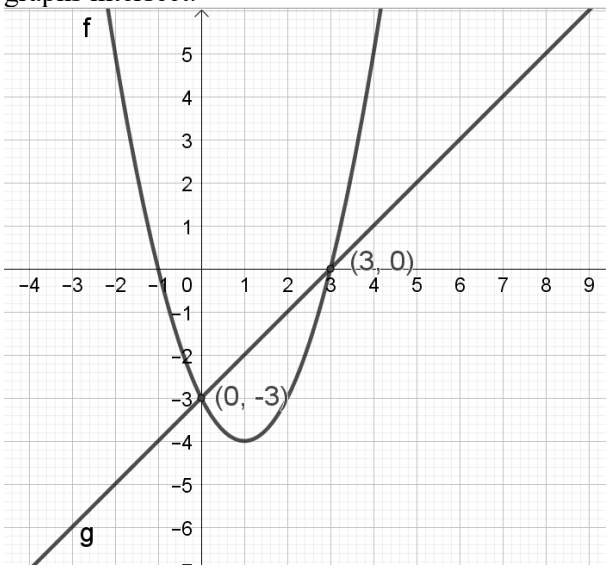
- Solve it graphically, by drawing both graphs to scale and find the points of intersection or
- Solve it algebraically.

In this lesson we will explain how to solve the equations algebraically.

EXAMPLE 4

Solve for x and y simultaneously

4.1	$y = x^2 - 1 \ and \ y - x = 5$ **Solution** $y - x = 5$ $\therefore y = x + 5$ $\boldsymbol{y = x^2 - 1}$ $\therefore \boldsymbol{x + 5} = x^2 - 1$ $\therefore 0 = x^2 - x - 6$ $\therefore 0 = (x - 3)(x + 2)$ $\therefore x = 3 \ or \ x = -2$ $y = x + 5$ For $x = 3$ for $x = -2$ $\boldsymbol{y = 3 + 5}$ $\boldsymbol{y = -2 + 5 = 3}$ $\boldsymbol{y = 8}$ $\boldsymbol{y = 3}$ The solution can also be written in the form $(3;\ 8)$ and $(-2;\ 3)$	**NOTE:** The first equation is a quadratic equation and the second equation is a linear equation. **Solution** STEP 1: make *y* the subject of the formula in the linear equation STEP 2: substitute $y = x + 5$ in the quadratic equation STEP3: write equation in standard form. STEP 4: then solve for *x* STEP 5: The corresponding values of y can be determined by substituting the values for x in the linear equation

4.2	Determine algebraically the points of intersection of the graphs indicated below $\boldsymbol{y = x^2 - 2x - 3 \; and \; y - x + 3 = 0}$ **Solution** $y = x^2 - 2x - 3$ ……. (equation 1) $y - x + 3 = 0$ …… (equation 2) $y = x - 3$ ……(equation 3) $x^2 - 2x - 3 = \boldsymbol{x - 3}$ $x^2 - 3x = 0$ $x(x - 3) = 0$ $\therefore x = 0 \; or \; x = 3$ $y = x - 3$ $y = 0 - 3$ *or* $y = 3 - 3$ $\therefore y = -3$ *or* $y = 0$ Points of intersection $(0, -3)$ and $(3;\ 0)$	STEP 1: make *y* the subject of the formula in the linear equation STEP 2: substitute $y = x - 3$ in the quadratic equation (equation 1) STEP3: write equation in standard form. STEP 4: then solve for *x* STEP 5: The corresponding values of y can be determined by substituting the values for x in the linear equation. (equation 3) **Note**: When we solve two equations simultaneously, we are looking for the ordered pairs that will satisfy both equations.	**Note**: When we solve two equations simultaneously, we are looking for the ordered pairs that will satisfy both equations. We can solve the equations of other pairs of graphs to find the points of intersection e.g. a straight line and a parabola, two parabolas or a straight line and a hyperbola etc. Remember we can represent each equation as a graph. If we draw graphs of both equations on the same set of axes, the solution of the simultaneous equation represents the points where the two graphs intersect. 

4.3	$3x - y + 2 = 0 \ldots\ldots\ldots\ldots\ (i)\ and$ $y = -x^2 + 2x + 8 \ldots\ldots\ldots (ii)$ $3x + 2 = y \ldots\ldots\ldots\ldots (iii)$ substitute (iii) into (ii) $\mathbf{3x + 2} = -x^2 + 2x + 8$ $0 = -x^2 - x + 6$ $0 = (x + 3)(x - 2)$ $x = -3 \quad or \quad x = 2$ $y = 3(-3) + 2 \quad or \quad y = 3(2) + 2$ $y = -7 \qquad\qquad y = 8$ $\therefore (-3; -7)\ and\ (2\,; 8)$	STEP 1: make *y* the subject of the formula in the linear equation. (In this case it will be more difficult to make x the subject of the formula therefore we choose y.) STEP 2: substitute $\boldsymbol{y = 3x + 2}$ in the quadratic equation STEP3: write equation in standard form. STEP 4: then solve for x STEP 5: The corresponding values of y can be determined by substituting the values for x in the linear equation

CAN YOU?

	Solve simultaneously for x and y in the following equations:	**Answers**
1.	$y = x^2 - 2\ and\ y - 2x = 1$	$x = 3\ or\ x - 1\ and\ y = 7\ or\ y = -1\ \ also\ \ (3; 7)\ or\ (-1;\ -1)$
2.	$y = 3x + 7\ and\ y = 2x^2 + 8$	$x = \frac{1}{2}\ \ or\ \ 1\ \ and\ y = \frac{17}{2}\ and\ 10\ \ also\ \ \left(\frac{1}{2}; \frac{17}{2}\right) ar\ (1;\ 10)$
3.	$x - 3y = 1\ \ and\ x^2 + xy + 9y^2 = 17$	$\left(-3;\ -\frac{4}{3}\right)$ or $(4; 1)$
4.	**Calculate the points of intersection of the graphs of:** $y = 2x \quad and\ y = 3x^2 - 1$	$\left(-\frac{1}{3}; -\frac{2}{3}\right)\ or\ (1; 2)$

Consolidation

- You should now be able to solve quadratic inequalities in one variable and interpret the solution graphically.
- You should also know how to solve equations in two unknowns, one of which is a linear and the other quadratic, algebraically.

ACTIVITY	**Mind Action Series**	**Via Africa**	**Siyavula**	Classroom Mathematics	Platinum
	Pg. 41 ex 9 Pg. 45 ex 11 and 12	Pg. 35 ex 9 Pg. 37 ex 10	End of chapter exercise Pg. 70 Pg. 76	Pg. 48 ex 2.14 Pg. 50 ex 2.5	Pg. 36 ex 8 and 9 Pg. 37 ex 10