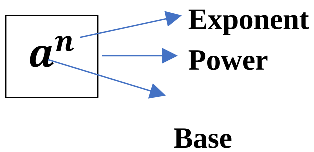
SUBJECT and GRADE	Mathematics Grade 11	
TERM 1	Week 1	
TOPIC	Exponents and Surds	
AIMS OF LESSON	• Defining a rational exponent. • Simplifying expression • Solving exponential equations. • Defining surds • Identifying complex, simple and mixed surds • Simplifying surds • Solving equations containing surds.	
RESOURCES	***Paper based resources***	***Digital resources***
	Please go to EXPONENTS AND SURDS chapter in your textbook.	Where you see this icon in the lesson you can click on it to see a video on concepts and calculations of EXPONENTS AND SURDS.

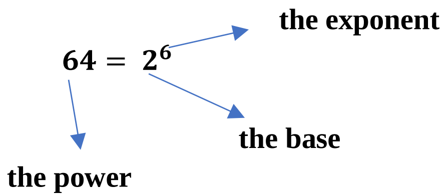
INTRODUCTION

Dear learner in this chapter we review the laws of exponents and exponential equations. When we've covered that, we will have a look at rational exponents and surds. You will also learn how to solve exponential equations, simplify surds and solve equations containing surds.

Concepts and skills

Exponents: The exponent of a number tells us how many times to multiply the number (the base) by itself.





$a^n = a \times a \times a \times \ldots\ldots until\ there\ are\ n\ factors\ of\ a, where\ a \in R\ and\ n \in R$

$a^2 = a \times a$

$2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

A) **LAWS of EXPONENTS**

These laws assume that **a** and **b** are positive real numbers

https://youtu.be/iTt3TVjsjuY

Laws of exponents only apply to multiplication, division, brackets and roots **and not to** addition and subtraction

No	**Algebraic Notation**	**Exponential Notation**	**Exponential laws in operation**
1	$16 = 2 \times 2 \times 2 \times 2$	$16 = 2^4$	When we multiply the same bases we ADD the exponents $\boldsymbol{a^n \times a^m = a^{n+m}}$
2	$\frac{64}{16} = 4$	$\frac{2^6}{2^4} = 2^2$	When we divide the same bases we minus the exponents (top minus bottom) $\boldsymbol{\frac{a^m}{a^n} = a^{m-n}}$
3	$4^3 = 64$	$\boldsymbol{(2^2)^3 = 2^6}$	When we have the exponents outside the brackets we distribute them into the brackets (exponent on the outside is multiplied by the exponent(s) on the inside) $\boldsymbol{(a^m)^n = a^{mn}}$
4	$4 \times 9 = 36$	$2^2 \times 3^2 = 6^2$	When we have non-identical bases, but identical exponents, we keep the exponents and multiply the bases (this same rule will also apply for division) $\boldsymbol{(ab)^n = a^n b^n}$
	$\sqrt{2} \times \sqrt{3} = \sqrt{6}$	$2^{\frac{1}{2}} \times 3^{\frac{1}{2}} = 6^{\frac{1}{2}}$	
5	$\left(\frac{4}{2}\right)^3 = \frac{64}{8} = 8$	$\left(\frac{4}{x^2}\right)^3 = \frac{4^{1\times3}}{x^{2\times3}} = \frac{64}{x^6}$	When a fraction is raised to a power, both the numerator and denominator are raised to the power $\boldsymbol{\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}}$

Exponential definition :

$x^{-n} = \frac{1}{x^n}$	This definition allows us to move numbers or variables from the top to the bottom of a fraction, or bottom to top	$\frac{3}{x^{-2}} = 3x^2$ or $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$
$x^0 = 1, \quad x \neq 0$	Any base (except zero)to the power of zero is equal to 1	$10^0 = 1$; $(2ab^2)^0 = 1$ NB $-10^0 = -1$ *but* $(-10)^0 = 1$

Study and work through each concept above as to ensure that you understand the laws and exponential definitions.
We will now do some Gr 10 revision to recap our content knowledge and prepare us for new added content of Exponents and Surds

Worked Example 1

Let us work through a few examples of how to apply these laws

https://youtu.be/Hn_rbQ2zSAU

Simplify without using a calculator

1. $y^3 \times y$ **2.** $9^6 \div 9^4$ **3.** $(y\,x^2)^7$ **4.** $\left(\frac{2x^3}{8y^{-4}}\right)^{-3}$ **5.** $(-3a^2)^2(2a)^3$

Solutions [Can you identify all the exponent laws and definitions?]

1. $y^3 \times y$

$= y^3 \times y^1$

$= y^4$

2. $9^6 \div 9^4$

$=9^{(6-4)}$

$=9^2$

81

3. $(y\,x^2)^7$

$=y^{(1\times7)}x^{(2\times7)}$

$=y^7x^{14}$

4. $\left(\frac{2x^3}{8y^{-4}}\right)^{-3}$

$=\left(\frac{8y^{-4}}{2x^3}\right)^3$

$=\left(\frac{4}{x^3y^4}\right)^3$

$=\frac{4^3}{x^{3\times3}y^{4\times3}} = \frac{64}{x^9y^{12}}$

5. $(-3a^2)^2(2a)^3$

$=9a^4.8a^3$

$=72a^7$

Can you?

Apply the laws to simplify the following without the use of a calculator

1. $y^{2n}.y^5$ 2. $a^{3n} \div a^n$ 3. $(x^2)^4$ 4. $\left(\frac{2x^2}{3y^3}\right)^3$ 5. $(-3y^3)^2$

Answers:

1. y^{2n+5} 2. a^{2n} 3. x^8 4. $\frac{8x^6}{27y^9}$ 5. $9y^6$

Rational numbers include all numbers that can be written as fractions in the form $\frac{a}{b}$ $where\ a, b \in Z\ but\ b \neq 0$

B) Simplifying expressions involving rational exponents

When compound numbers are raised to a rational exponent, it can be simplified by using prime factors and the rule for raising a power to another power e.g. $\mathbf{32^{\frac{3}{5}} = (2^5)^{\frac{3}{5}} = 2^3 = 8}$

Consider the following $\sqrt[n]{a} = a^{\frac{1}{n}}$ if we raise both sides to the power *m* we have

$$(\sqrt[n]{a})^m = (a^{\frac{1}{n}})^m$$

Therefore $\sqrt[n]{a^m} = a^{\frac{m}{n}}$

Work through all the calculations in the worked examples and identify laws and definitions

Worked examples 2

Simplify the following expressions, without a calculator and leave your answers with positive exponents.

1. $(8)^{\frac{1}{3}}$ 2. $(\frac{1}{4})^{-2}$ 3. $(8a^6b^{12})^{\frac{1}{3}}$ 4. $\sqrt[5]{32^2}$ 5. $(\frac{-2x^{-2}}{(-2x)^{-2}})^{-\frac{1}{3}}$

Solutions

1. $(8)^{\frac{1}{3}}$
   $=(2^3)^{\frac{1}{3}}$
   $= 2$

2. $(\frac{1}{4})^{-2}$
   $= 4^2$
   $= 16$

3. $(8a^6b^{12})^{\frac{1}{3}}$
   $= (2^3a^6b^{12})^{\frac{1}{3}}$
   $= 2a^2b^4$

4. $\sqrt[5]{32^2}$
   $= \sqrt[5]{(2^5)^2}$
   $=(2^5)^{\frac{2}{5}}$
   $= 2^2 = 4$

5. $(\frac{-2x^{-2}}{(-2x)^{-2}})^{-\frac{1}{3}}$
   $=(\frac{-2x^{-2}}{(-2)^{-2}\,x^{-2}})^{-\frac{1}{3}}$
   $=(-2x^{-2}.(-2)^2x^2)^{\frac{-1}{3}}$
   $= (-2^3)^{\frac{-1}{3}}$
   $=-2^{-1} = -\frac{1}{2}$

CAN YOU simplify the following expressions, without the use of a calculator? Your answers should have positive exponents.

1. $\left(\frac{1}{2}\right)^{-3}$ **2.** $3^{-1}.2^0$ **3.** $\left(\frac{\sqrt{x}}{x^{\frac{-3}{2}}}\right)^{\frac{1}{2}}$ **4.** $\left(3\,\frac{3}{8}\right)^{\frac{-2}{3}}$ **5.** $\sqrt[3]{\frac{27a^3b^6}{64c^9}}$

Answers

1. **8** **2.** $\frac{1}{3}$ **3.** x **4.** $\frac{4}{9}$ **5.** $\frac{3ab^2}{4c^3}$

C. Simplifying expressions using factors

The questions that you would need to solve can be divided into one of the two types

Type 1: How to simplify expressions consisting of one term

When the expression consists of one term (only multiplication and division) factorise into prime factors and apply the exponential laws.

Follow and understand the following steps

1. Rewrite all composite numbers as a product of their prime factors.
2. Use exponential laws to raise all powers appropriately.
3. Use the rule for division to move all bases into the numerator
4. Then simplify

$$\frac{a \rightarrow \ Numerator}{b \ \rightarrow denominator}$$

Worked example 3

Simplify:

$$\frac{25^n.36^{n+1}}{81.\ 30^{2n}}$$

Factorise into prime numbers

$$\frac{(5^2)^n.(2^2.3^2)^{n+1}}{3^4.(2.3.5)^{2n}}$$

Apply exponent laws

$$\frac{5^{2n}.2^{2n+2}\ .3^{\ 2n+2}}{3^4.2^{2n}3^{2n}.5^{2n}}$$

*

$$\mathbf{2^{2n+2-2n}.3^{2n+2-4-2n}}$$

$$\mathbf{2^2.3^{-2}}$$

Remember $3^{-2} = \frac{1}{3^2}$

$$\mathbf{\frac{4}{9}}$$

Can you ?

Simplify the following expressions

1. $\frac{8^{n+3}.32^{-n-1}.6^{2n}}{9^n}$

2. $\frac{2.3^{x-1} - 3^x}{3^x - 3^{x-1}}$

3. **Write down the values of a, b and c**

$64^a = 8$ $\quad$ $64^b = 4$ $\quad$ $64^c = 2$

Type 2: How to simplify expressions consisting of more than one term

When an expression consists of more than one term you have to factorise the expression before you can simplify (terms are separated by + or – signs)

Follow and understand the following steps

1. Separate and factorise
2. Simplify further using your knowledge of common fractions

Worked example 4

Simplify: $3^{x+1} + 3^x$ $\quad$ [the common factor in each term is 3^x]

$=3^x(3 + 1)$ $\quad$ Divide by the common factor

[$3^{x+1} \div 3^x = 3$ and $3^x \div 3^x = 1$]

$=3^x.4$

Worked example 5

$$\frac{2^{x+3} - 2.2^x}{2^x}$$

$$\frac{2^x.2^3 - 2.2^x}{2^x}$$

$$\frac{2^x(8 - 2)}{2^x} = 6$$

Answers

1. 16 $\quad$ 2. $\frac{15}{2}$

3 $\quad$ a = $\frac{1}{2}$ $\quad$ b = $\frac{1}{3}$ $\quad$ c = $\frac{1}{6}$

D. Exponential Equations

There are two types of exponential equations:

- **Type 1 :** Equations with the exponent as the unknown
- **Type 2:** Equations with the base as the unknown

Type 1: Variables is in an exponent

Equations with one term on either side of the equation

This type of equation is using the basic premise that if $\boldsymbol{a^x = a^b, then\ x = b}$, **for** $\boldsymbol{a \neq 0}$

For example, consider the equation $3^x = 9$. The equation can be solved as follows:

$3^x = 9$

$\therefore 3^x = 3^2$ write 9 to the base of 3

$\therefore x = 2$ Equate the exponents

The intention is therefor to express both sides of the equation with the same base so that we can equate the exponents.

Worked examples 6

If $\boldsymbol{x \in R, solve\ for\ x\ in\ the\ following\ equations}$

1. $4^{x-1} = 8^{-1}$ 2. $(5^{x-2})^x = 125$

Study and work through this worked examples step by step until you understand each calculation.

Solutions

1. $4^{x-1} = 8^{-1}$

$(2^2)^{x-1} = (2^3)^{-1}$ express each term as a product of its prime factors

$2^{2x-2} = 2^{-3}$

$2x - 2 = -3$ equate the exponents

$2x = -1$

$\therefore x = -\frac{1}{2}$

2. $(5^{x-2})^x = 125$

$5^{x^2-2x} = 5^3$ express each term as a product of its prime factors

$\therefore x^2 - 2x = 3$ equate the exponents

$x^2 - 2x - 3 = 0$ factorise to solve the trinomial

$(x-3)(x+1) = 0$

$\therefore x = 3\ or\ x = -1$

CAN YOU solve for x in the following equations?

1. $9^{x+1} = 27^x$
2. $\left(\frac{1}{2}\right)^{x^2+9} = 4^{x-4}$
3. $4 . 3^{7x} = 9.2^{7x}$

Answers

1. $\boldsymbol{x = 2}$
2. $\boldsymbol{x = -1}$
3. $\boldsymbol{x = \frac{2}{7}}$

Equations with more than one term on each side of the equation

Type 1: The exponent is x: collect the terms with an exponent of x on the LHS ,and write the constant on the RHS. Take out the common factor from the LHS.

Type 2 If one of the terms has an exponent of $2x, e.g. 3^{2x},$ this indicates a quadratic equation. Write in standard form, and equate to zero. Factorise and set each of the factors equal to zero and solve

Worked examples 7

Solve for x if $\boldsymbol{x \in R}$

1. $3^x - 3^{x-2} = 24$

$3^x(1 - 3^{-2}) = 24$ Factorise using common factors

$3^x\left(\frac{8}{9}\right) = 24$ Simplify

$3^x = 24 \times \frac{9}{8}$

$3^x = 27$ Express each term as a product of its prime factors

$3^x = 3^3$ equate the exponents once the bases are equal

$\therefore x = 3$

3. Solve for *x*

$2^{x+1} + 2^3.2^{-x} = 17$ Factorise the LHS

$2^x.2^1 + 2^3.\left(\frac{1}{2^x}\right) = 17$ write $2^{-x} as\ \frac{1}{2^x}$

$k.2^1 + 2^3\left(\frac{1}{k}\right) = 17$ Let $2^x = k\ and\ multiply\ by\ k\ throughout$

$2k^2 + 8$= 17k

$2k^2 - 17\,k + 8 = 0$ factorise the quadratic equation

$(2k - 1)(k - 8) = 0$

$k = \frac{1}{2}$ or $k = 8$ Solve for *k*

$2^x = 2^{-1}\ or\ 2^x = 2^3$ Solve for *x*

$\therefore x = -1\ or\ x = 3$

In the previous examples the equation only had one solution. However we will now look at equations that has 2 solutions.

When solving exponential equations we can generalize this as follows

If $x^{\frac{m}{n}} = c\ , where\ c\ is\ any\ constant\ then$:

- If *m* is odd , then there is only one solution.
- If *m* is even, then there are two solutions, one positive and one negative

2. $5^{2x} - 4.5^x - 5 = 0$ [the coefficients of the exponents of *x* differs]

Pay careful attention to the following steps

Let $5^x = k, then\ 5^{2x} = k^2$

Factorise the quadratic equation

Solve for *k*

Then solve for *x*

$$k^2 - 4k - 5 = 0$$
$$(k - 5)(k + 1) = 0$$
$$k = 5\ or\ k = -1$$
$$5^x = 5^1\ or\ 5^x = -1$$
$$\therefore x = 1$$

Since there is no value of x that will give us a negative value in the second solution, this solution is invalid. Therefor ,x=1 is the only valid solution

CAN YOU solve for *x* if $\boldsymbol{x \in R}$

1. 4^{x+1} −64= 0
2. $3.3^{2x} - 4.3^x = -1$

Answer

1. $\boldsymbol{x = 2}$ **2.** $\boldsymbol{x = -1\ or\ 0}$

Type 2: Variables is in the base

- Surds that have even roots (i.e. the denominator is even) have two possible answers so…

  $16^{\frac{3}{2}} = \sqrt[2]{16^3} = \pm 16^{\frac{3}{2}}$
- Surds that have odd roots (i.e the denominator is odd) only have one possible answer so…

  $16^{-\frac{2}{3}} = \sqrt[3]{64^{-2}}$

If the unknown variable (say *x)* is a base, we raise both sides to the same power (the reciprocal of the power of x) in order to change the exponent of the unknown to 1

Worked example

Solve for $x \ if \ x \in R$

1. $2x^{\frac{2}{3}} = 32$ [first divide by 2, which is the coefficient of x]

   $x^{\frac{2}{3}} = 16$

   $\left(x^{\frac{2}{3}}\right)^{\frac{3}{2}} = \pm(16)^{\frac{3}{2}}$ [raise both sides to the reciprocal power]

   $x = \pm(2^4)^{\frac{3}{2}}$ [express 16 as a product of its prime factors]

   $x = \pm 2^6$

   $\therefore x = \pm 64$

2. $\boldsymbol{x^{-\frac{3}{2}} = 64}$

   $\boldsymbol{\left(x^{-\frac{3}{2}}\right)^{-\frac{2}{3}} = (64)^{-\frac{2}{3}}}$ Please note that the reciprocal power must also be negative

   $\boldsymbol{x = (2^6)^{-\frac{2}{3}}}$

   $\boldsymbol{x = 2^{-4}}$

   $\boldsymbol{\therefore = \frac{1}{16}}$

CAN YOU?

1. Solve for x if $3x^{\frac{5}{2}} = 96$
2. Solve for x if $3x^{-\frac{5}{3}} + 16 = 112$

Answers

1. $x = 4$
2. $x = 2^{-3} \quad = \frac{1}{8}$

E) Simplification of Surds

https://youtu.be/hcsHHWvNZWo

- Definition: a surd is the root of a whole number that produces an irrational number.
- Therefor a surd is the root of a number that cannot be determined exactly.
- An irrational number is a number that cannot be expressed as an integer or as a fraction that results in a finite number of digits (i.e. is a number that is non- recurring, non-terminating decimal)
- Examples would be $\sqrt{3},\ \sqrt{7}\ \ \sqrt[3]{5}$
- If $\sqrt[n]{a} = x, then\ x^n = a$
- If $\sqrt[n]{a} = a^{\frac{1}{n}}$

Laws of surds	examples	Explanatory notes
$\sqrt[n]{x} \times \sqrt[n]{y} = \sqrt[n]{x.y}$	• $\sqrt{5} \times \sqrt{3} = \sqrt{15}$ • $\sqrt{20} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$	When surds are multiplied they can be split apart and rooted individually $\sqrt{20}$ is not in its simplest form because a perfect square is a factor of 20 (4)
$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$	• $\sqrt{\frac{16}{25}} = \frac{\sqrt{16}}{\sqrt{25}} = \frac{4}{5}$ • $\frac{\sqrt{3}}{\sqrt{18}} = \sqrt{\frac{3}{18}} = \sqrt{\frac{1}{6}}$	When surds are divided they can be split apart and rooted individually
$a\sqrt{c} \pm b\sqrt{c} = (a \pm b)\sqrt{c}$	• $5\sqrt{6} - 2\sqrt{6} = 3\sqrt{6}$	If the root is of the same number or variable surds are treated the same as like terms. It can be added or subtracted, and the root remains the same.
$\sqrt[mn]{x} = \sqrt[m]{\sqrt[n]{x}}$	• $\sqrt[6]{64} = \sqrt[3\times 2]{64} = \sqrt[3]{\sqrt{64}} = \sqrt[3]{8} = 2$ • $\sqrt{\sqrt[3]{5^2}} = \sqrt[6]{5^2} = 5^{\frac{2}{6}} = 5^{\frac{1}{3}}$	When taking the root of a root, it is the same as, taking the single root to the product of both roots.

Using the laws and definitions of surds

As with exponential problems, you will lose marks if you do not show your workings, including writing composite numbers as powers of prime numbers or as prime factors (only use your calculator to check your answers.)

Worked examples (Multiplication and Division of surds)

Simplify the following expressions without using a calculator. Show all your calculations.

1. $\left(\sqrt{4}\right)^2 =$ **2** $\frac{\sqrt[3]{16}}{\sqrt[3]{2}} =$ **3.** $\frac{\sqrt{12}\times\sqrt{24}}{\sqrt{8}} =$

4. Determine, without the use of the calculator, which is greater $\sqrt{7}$ or $\sqrt[3]{15}$

Work through each worked example and apply the multiplication and division law of surds

$\sqrt[n]{a}.\sqrt[n]{b} = \sqrt[n]{a.b}$ and $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Solutions

1. $\left(\sqrt{4}\right)^2 = \sqrt{4}\times\sqrt{4} = \sqrt{16}=4$

2. $\frac{\sqrt[3]{16}}{\sqrt[3]{2}} = \sqrt[3]{\frac{16}{2}} = \sqrt[3]{8} = 2$

3. $\frac{\sqrt{12}\times\sqrt{24}}{\sqrt{8}}$

$= \frac{\sqrt{4}\times\sqrt{3}\times\sqrt{4}\times\sqrt{6}}{\sqrt{4}\times\sqrt{2}}$

$= \frac{2\sqrt{3}\times2\sqrt{6}}{2\sqrt{2}}$

$= \frac{4\sqrt{18}}{2\sqrt{2}}$

$= 2\sqrt{9} = 2\times3 = 6$

4. $\sqrt{7} = 7^{\frac{1}{2}} = 7^{\frac{3}{6}} = \sqrt[6]{7^3} = \sqrt[6]{343}$

$\sqrt[3]{15} = 15^{\frac{1}{3}} = 15^{\frac{2}{6}} = \sqrt[6]{15^2} = \sqrt[6]{225}$

Express all as $\sqrt[6]{\ }$

$\sqrt[6]{343} > \sqrt[6]{225}$

$\therefore \sqrt{7} > \sqrt[6]{15}$

CAN YOU simplify the following expressions?

1. $2\sqrt{3}.5\sqrt{3}$ **2.** $\left(3\sqrt{2}+3\right)\left(3\sqrt{2}-3\right)$ **3.** $\frac{\sqrt{50}+\sqrt{18}}{\sqrt{32}}$ **4.** $\sqrt{21}.\sqrt{60}.\sqrt{35}$ **5.** Arrange in descending order $\sqrt[3]{5}$; $\sqrt{3}$; $\sqrt[6]{26}$

Answers

1. 30 **2.** 9 **3.** 2 **4.** 210 **5.** $\sqrt{3}$; $\sqrt[6]{26}$; $\sqrt[3]{5}$

Worked example (Addition and subtraction of surds)

1. $4\sqrt{2}+\sqrt{3}-\sqrt{2}-2\sqrt{3}$ **2.** $\frac{\sqrt{48}-\sqrt{3}}{\sqrt{27}}$ **3.** $2\sqrt{8}-4\sqrt{32}+3\sqrt{50}$

Solutions

1. $4\sqrt{2}+\sqrt{3}-\sqrt{2}-2\sqrt{3}$ [Remember only like surds can be added or subtracted … Think of it as $4x+y-x-2y=3x-y$]]

$= 3\sqrt{2}-\sqrt{3}$

2. $\frac{\sqrt{48}-\sqrt{3}}{\sqrt{27}}$

$= \frac{\sqrt{16\times3}-\sqrt{3}}{\sqrt{9\times3}}$ [write each number as a product of a square number and another number]

$= \frac{4\sqrt{3}-\sqrt{3}}{3\sqrt{3}}$ [use the surd laws to separate the square roots]

$= \frac{3\sqrt{3}}{3\sqrt{3}} = 1$

3. $2\sqrt{8} - 4\sqrt{32} + 3\sqrt{50}$

$= 2\times2\sqrt{2} - 4\times4\sqrt{2} + 3\times5\sqrt{2}$ simplify surds using prime factors

$= 4\sqrt{2} - 16\sqrt{2} + 15\sqrt{2}$ Add and subtract like terms

$= 3\sqrt{2}$

CAN YOU simplify the following surds?

1. $\sqrt{7} + 2\sqrt{7}$- $6\sqrt{7}$
2. $\frac{\sqrt{75}+\sqrt{48}}{\sqrt{12}}$
3. $\sqrt{50}\left(\sqrt{18}+\sqrt{32}\right)$

Answers

1. $-3\sqrt{7}$ **2.** $4\frac{1}{2}$ **3.** 70

Note to remember

$\sqrt{8} = \sqrt{4\times2} = 2\sqrt{2}$

$\sqrt{32} = \sqrt{16\times2} = 4\sqrt{2}$

$\sqrt{50} = \sqrt{25\times2} = 5\sqrt{2}$

https://youtu.be/w3PGLLT5nr4

F) Rationalising the denominator

The principal of rationalising the denominator is to remove the surds(irrational numbers)in the denominator.
It is generally accepted that an answer is not completely simplified if there is a root/ surd in the denominator.

$\frac{a}{\sqrt{b}} = \frac{a}{\sqrt{b}}\times\frac{\sqrt{b}}{\sqrt{b}} = \frac{a\sqrt{b}}{b}$ To rationalize the denominator, the aim is to multiply by a version of 1 that will remove the surd from the denominator.

Remember $\sqrt{a}\times\sqrt{a} = a$

Worked Example

1. $\frac{5}{\sqrt{11}}$

$= \frac{5}{\sqrt{11}}\times\frac{\sqrt{11}}{\sqrt{11}}$

$= \frac{5\sqrt{11}}{11}$

2. $\frac{3}{2+\sqrt{2}}$

$= \frac{3}{2+\sqrt{2}}\times\frac{2-\sqrt{2}}{2-\sqrt{2}}$

$= \frac{3(2-\sqrt{2})}{(2+\sqrt{2})(2-\sqrt{2})}$

$= \frac{6-3\sqrt{2}}{2}$

CAN YOU?

Rationalise the denominator of:

1. $\frac{6}{\sqrt{3}}$
2. $\frac{5}{4-\sqrt{3}}$

Answers: 1. $2\sqrt{3}$ 2. $\frac{20+5\sqrt{3}}{13}$

F) Surd Equations

Solving equations involving surds:

- To solve this type of equation we use the basic premise that $\left(\sqrt{x}\right)^2 = x$
- To solve equations with surds, rearrange the equation so that the term containing the surd (or root sign: $\sqrt{\quad}$) is on its own on one side of the equation, otherwise the middle term, when squaring, will again contain a surd.
- The validity of all possible solutions MUST be checked by substitution into the original equation.
- From gr 10 content we know that :if $x \geq 0$ then the expression $\sqrt{x}$ will be a real number.
- For example $\sqrt{16}; \sqrt{5}; -\sqrt{6}$ are r**ea**l numbers since the numbers under the square root signs are positive.
- However, if $x < 0$, then the expression $\sqrt{x}$ will be a **non-real** number
- For example , $\sqrt{-1}$; $\sqrt{-4}$; $\sqrt{-16}$ are non- real numbers since the numbers under the square root signs are negative.
- If a particular solution does not satisfy the original equation, it is not a solution.

Worked examples

1. Solve for x in $\sqrt{2x-4} + x = 6$

Solution

$\sqrt{2x-4} + x = 6$

$\sqrt{\mathbf{2x-4}} = 6 - x$ [Isolate the surd term]

$\left(\sqrt{2x-4}\right)^2 = (6-x)^2$ [square both sides of the equation]

$2x - 4 = 36 - 12x + x^2$

$0 = 36 + 4 - 12x - 2x + x^2$

$x^2 - 14x + 40 = 0$ [solve the equation]

$(x-10)(x-4) = 0$

$\therefore x = 10 \text{ or } x = 4$

Work through the worked example until you are confident that you understand **and know** why and how to do each calculation.

CAN YOU?

1. Solve for x in $\sqrt{x+2} - x = 0$
2. Solve for x in $\sqrt{3x+4} = 2x + 3$

Answers:

1. $x = -\frac{5}{4}$ or $x = -1$
2. $x = -2$

Check the answers for validity

If x=10

LHS $\sqrt{2(10)-4}+10$

$\sqrt{16}+10$

=14

$\neq 6$

$\therefore LHS \neq RHS$

$x = 10$ *is not a solution*

If $x = 4$

LHS $\sqrt{2(4)-4}+4$

$=\sqrt{4}+4$

=6

$\therefore LHS = RHS$

$\therefore x - 4$ is the only solution

Note: The key to solving all equations with surds is to isolate the surd then square both sides to remove the root sign. **But** by squaring both sides of the equation, you may have introduced an extra solution which might be invalid. Therefor, you must check all the solutions obtained in the original equation.

The square root of a negative number is non- real (does not exist) and neither can the square root of a number be negative

Consolidation

- Remember to revise the number systems as to ensure that you know all the different types of numbers.
- Exponents and Surds is part of Algebraic expressions which counts about 30% of the final Paper 1 examination
- A sound knowledge of exponents will assist you in Calculus in Grade 12.
- Surd equations need to be covered thoroughly – learners must test whether their solutions satisfy the original equation
- Practice by working out old question papers to get acquainted with the way question are asked in exams.

ACTIVITY	**Mind Action Series** Revision exercise pg 18	**Via Africa** Summary and questions pg. 18 -19	**Siyavula** End of chapter exercises Pg 16 and 17	Classroom mathematics Revision and consolidation Pg 22	Platinum Revision Pg 20 - 21