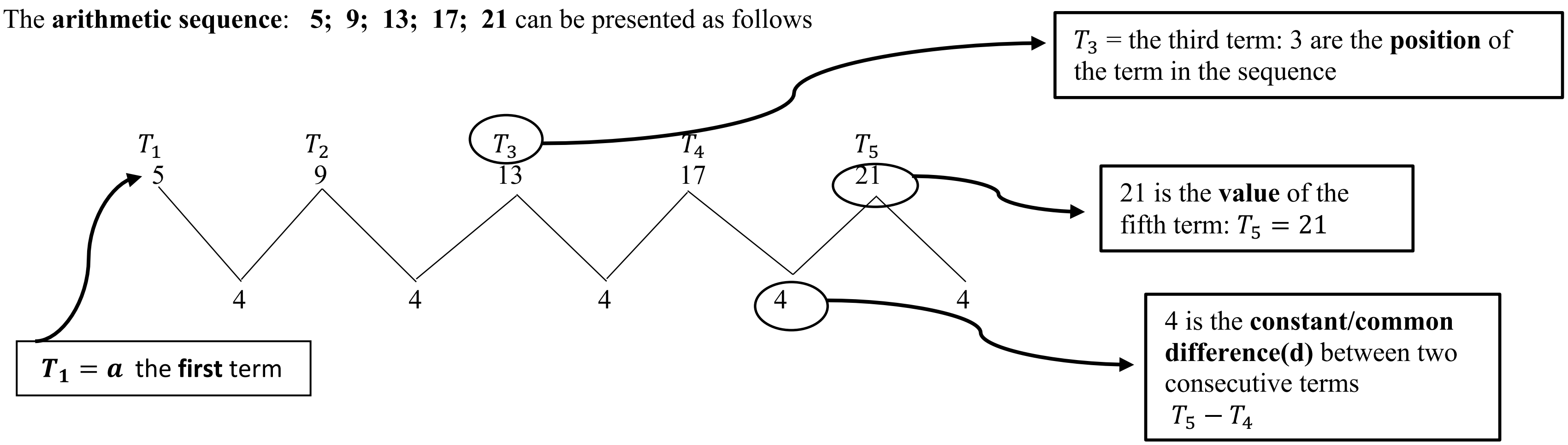
SUBJECT and GRADE	Mathematics Grade 12	
TERM 1	Week 1	
TOPIC	Sequences and Series	
AIMS OF LESSON	• Recognise an arithmetic Sequences • Find the general arithmetic sequence • Answer question based on the arithmetic sequence like finding the position in a sequence. • Find the sum of an arithmetic sequence • Sigma notation	
RESOURCES	***Paper based resources***	***Digital resources***
	Textbook chapter about Sequences and Series	https://www.youtube.com/watch?v=WE3S6OAwc-s
INTRODUCTION	In the previous grades you were introduced to numeric number patterns which is a sequence of numbers that follow a specific pattern. An example of a **linear pattern (arithmetic sequence)** is one where there is a **constant difference between consecutive terms.** In other words, the **same number** will be added to, or subtracted from each consecutive term	

A sequence is a ordered list of numbers or objects. A linear number pattern is also called an **ARITHMETIC SEQUENCE**

The **arithmetic sequence**: **5; 9; 13; 17; 21** can be presented as follows



An **Arithmetic sequence** is a sequence where the **difference between consecutive numbers terms remains constant** i.e.

In the sequence: **5; 9; 13; 17; 21; …**

$\boldsymbol{a}$ = **the first term** $= \boldsymbol{T_1}$

$\boldsymbol{d}$ = **constant difference**

$\boldsymbol{d} = T_2 - T_1 = T_3 - T_2$

$n = number\ of\ terms$

We see that $\boldsymbol{a = 5}$ and $\boldsymbol{d = 4}$

$\boldsymbol{T_1} = 5 = \qquad \boldsymbol{a}$

$\boldsymbol{T_2} = 9 = 5 + 4 = \qquad \boldsymbol{a + d}$

$\boldsymbol{T_3} = 13 = 5 + 2(4) = \boldsymbol{a + 2d}$

$\boldsymbol{T_4} = 17 = 5 + 3(4) = \boldsymbol{a + 3d}$

$\boldsymbol{T_n} = 5 + (n-1)(4) = \boldsymbol{a + (n-1)d}$

The **General Term** $\boldsymbol{T_n}$

Is given by

$$\boldsymbol{T_n = a + (n-1)d}$$

Example 1: Given the sequence: 2; 5; 8; …	Solution: **Constant difference:** $\boldsymbol{d = T_2 - T_1 = 5 - 2 = 3}$ $\boldsymbol{a = T_1 = 2}$
a) Determine the general term of the sequence.	$T_n = a + (n-1)d$ $= 2 + (n-1)3$ $= 2 + 3n - 3$ $= 3n - 1 \qquad \therefore T_n = 3n - 1$
b) Use the general rule to determine the 40th term.	$T_n = 3n - 1$ $\therefore T_{40} = 3(40) - 1$ ← The **position** of the term is 40. Therefore, $n = 40$ $T_{40} = 119$
c) Which term in the pattern will be equal to 2012.	$T_n = 2012$ $\therefore T_n = 3n - 1 = 2012$ ← The **value** of the term is 2012. Therefore, $n = 671$ $3n = 2012 + 1$ $3n = 2013$ $n = 671 \qquad \therefore$ term number 671 of the sequence is equal to 2012

Example 2: Find the number of terms in the arithmetic sequence -2; -6; -10; … ; -150	Solution: $\boldsymbol{d} = -6-(-2) = \mathbf{-4}$ $T_n = a+(n-1)d$ $T_n = -2+(n-1)(-4)$ $T_n = -4n+2$ $T_n = -4n+2 = \mathbf{-150}$ $\therefore -4n = -152$ $\therefore n = 38$ There is 38 terms Constant difference: $\boldsymbol{d} = \mathbf{10}$ $\boldsymbol{T_4} = \mathbf{39}$	Example 3: Determine the first three terms of an arithmetic sequence if the constant difference is 10 and the fourth term is 30.	Solution: Constant difference: $\boldsymbol{d} = \mathbf{10}$ $\boldsymbol{T_4} = \mathbf{39}$ $T_n = a+(n-1)d$ $T_4 = a+(4-1)d$ $39 = a+3d$ $39 = a+3(10)$ $9 = a$ Hence the sequence is 9; 19; 29
Example 4: In an arithmetic sequence the 2^{nd} term is 9 and the 5^{th} term is 21. Determine a) The first three terms of the sequence. b) The 60^{th} term	$T_2 = a+d = 9$ (1) 2^{nd} term $T_5 = a+4d = 21$ (2) 5^{th} term $3d = 12$ **(2) − (1)** $\boldsymbol{d} = \mathbf{4}$ $\therefore T_2 = a+d = 9$ $a+4 = 9$ $\boldsymbol{a} = \mathbf{5}$ First three terms are 5; 9; 13; $T_{60} = a+59d = 5+59(4) = 241$ Hence the 60^{th} term = 241	Example 5: $2p-3;\ 3p-1;\ 5p-2$ are the first three terms of an arithmetic sequence. a) Determine the value of p. b) The first three terms of the sequence. c) Determine the term equal to 2013	Solution: a) $\boldsymbol{d} = T_2 - T_1 = T_3 - T_2$ $(3p-1)-(2p-3) = (5p-2)-(3p-1)$ $3p-1-2p+3 = 5p-2-3p+1$ $p+2 = 2p-1$ $\boldsymbol{p} = \mathbf{3}$ b) Replacing $p = 3$ in the sequence we have the first three terms as 3; 8; 13 c) $T_n = a+(n-1)d = 2013$ $3+(n-1)(5) = 2013$ $3+5n-5 = 2013$ $5n = 2013$ $n = 403$
CAN YOU?	1) Given the following sequence: 3; 8; 13; 18; … Determine: a) The general term b) The 20th term c) Which term of the sequence is equal to 223 2) In an arithmetic sequence, $T_3 = -2$ and $T_8 = 23$. Determine the first term and the constant difference. 3) Find the number of terms in the arithmetic sequence $-5;\ -11;\ -17;\ \ldots;\ -491$ 4) The first three terms of an arithmetic sequence is $x-8;\ x;\ 2x-5$. Determine a) The value of x. b) The general term. c) The value of the 115^{th} term.		Solutions: 1) a) $T_n = 5n-2$ b) 98 c) $n = 45$ 2) $d = 5$ $a = -12$ 3) 82 4) a) 13 b) $T_n = 8n-3$ c) 917

SERIES: A series is created by **adding the terms of a sequence.**

2; 5; 8; 11; 14; — **Arithmetic Sequence**

2 + 5 + 8 + 11 + 14 — **Arithmetic Series**

The **Sum of a sequence** is labelled as $\boldsymbol{S_n}$ or the Greek symbol $\sum$

In the series 2 + 5 + 8 + 11 + 14 + …

$$\boldsymbol{S_1} = T_1 = 2$$
$$\boldsymbol{S_2} = T_1 + T_2 = \quad \boldsymbol{S_1} + \boldsymbol{T_2} = 2 + 5 = 7$$
$$\boldsymbol{S_3} = T_1 + T_2 + T_3 = \quad \boldsymbol{S_2} + \boldsymbol{T_3} = 2 + 5 + 8 = 15$$
$$\boldsymbol{S_4} = T_1 + T_2 + T_3 + T_4 = \boldsymbol{S_3} + \boldsymbol{T_4} = 2 + 5 + 8 + 11 = 26$$

.

.

$$\boldsymbol{S_n} = \boldsymbol{S_{n-1}} + \boldsymbol{T_n}$$

$$\boxed{\boldsymbol{S_n} = \boldsymbol{S_{n-1}} + \boldsymbol{T_n}}$$

Let $\boldsymbol{T_n} = \boldsymbol{a} + (\boldsymbol{n} - \boldsymbol{1})\boldsymbol{d} = \boldsymbol{l}$ the last term

Then
$$S_n = a + (a+d) + (a+2d) + \dots + (l-d) + l$$
$$S_n = l + (l-d) + (l-2d) + \dots + (a+d) + a$$
$$2S_n = (a+l) + (a+l) + (a+l) + \dots + (a+l) + (a+l)$$
$$2S_n = n(a+l)$$
$$\boldsymbol{S_n} = \frac{\boldsymbol{a}}{\boldsymbol{2}}(\boldsymbol{a} + \boldsymbol{l})$$
$$S_n = \frac{a}{2}(a + \boldsymbol{a} + (\boldsymbol{n} - \boldsymbol{1})\boldsymbol{d}) = \frac{\boldsymbol{n}}{\boldsymbol{2}}[\boldsymbol{2a} + (\boldsymbol{n} - \boldsymbol{1})\boldsymbol{d}]$$

Hence the **sum of the first n terms** of an **arithmetic series** is given by the formule:

$$\boxed{\boldsymbol{S_n} = \frac{\boldsymbol{n}}{\boldsymbol{2}}[\boldsymbol{2a} + (\boldsymbol{n} - \boldsymbol{1})\boldsymbol{d}] \quad \textbf{or} \quad \frac{\boldsymbol{n}}{\boldsymbol{2}}[\boldsymbol{a} + \boldsymbol{l}]\ ;\ \boldsymbol{l} = \textbf{\textit{last term}}}$$

Example 6:

Consider the arithmetic series $(-1) + \left(\frac{-3}{2}\right) + (-2) + \cdots + (-16)$.

a) Determine the number of terms in this series.	b) Calculate the sum of the series.
Solution: $T_n = a + (n-1)d$ $T_n = -1 + (n-1)\left(-\frac{1}{2}\right) = -16$	Solution: $S_n = \frac{n}{2}[a + l]$

$\therefore\ 1 - \frac{1}{2}n + \frac{1}{2} = -16$

$\therefore\ -\frac{1}{2}n = -16 + \frac{1}{2}$

$\therefore\ n = 31$

$\therefore S_{31} = \frac{31}{2}[-1 + (-16)]$

$\therefore\ S_{31} = -\frac{527}{2}$

<u>Example 7:</u>

How **many terms** of the arithmetic series $1 + 4 + 7 + \cdots$ will **add up to 145.**

<u>Solution:</u>

$a = 1;\ d = 3;\ n = ?;\ S_n = 145$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$145 = \frac{n}{2}[2(1) + (n-1)3]$

$290 = n(2 + 3n - 3)$

$290 = n(3n - 1)$

$0 = 3n^2 - n - 290$

$0 = (3n + 29)(n - 10)$

$n = -\frac{29}{3}$ or $n = 10$

$\therefore n = 10\ ;\ n \epsilon N$

<u>Example 8:</u>

Consider the arithmetic series $-4 - 1 + 2 + \cdots$

Calculate the smallest value of n for which $S_n > 300$

<u>Solution:</u>

$a = -4;\ d = 3;\ n = ?;$ let $S_n = 300$

$S_n = \frac{n}{2}[2a + (n-1)d] = 300$

$\frac{n}{2}[2(-4) + (n-1)3] = 300$

$\therefore\ 3[3n - 11] = 600$

$\therefore\ 3n^2 - 11n - 600 = 0$

$\therefore\ n = \frac{-(11) \pm \sqrt{(-11)^2 - 4(3)(-600)}}{2(3)}$

$\therefore\ n = 16{,}09$ or $n = -12{,}43$

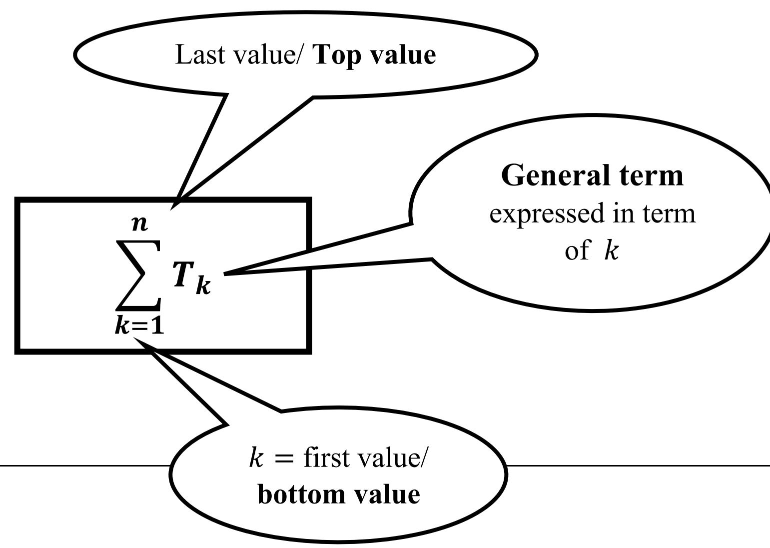
$\therefore$ The smallest possible value of n is 17.

SIGMA NOTATION: The Greek letter $\sum$ **Sigma means the sum of .**

$$\sum_{k=1}^{n} T_k = T_1 + T_2 + T_3 + \cdots + T_n = S_n$$

n =number of terms:

$$n = top - bottom + 1$$



$$S_n = \frac{n}{2}[2a + (n-1)d] = \sum_{k=1}^{n}[a + (k-1)d]$$

<u>Example 9:</u> Determine the value of $\sum_{n=1}^{5}(3n+2)$

<u>Solution:</u>
Substitute $n = 1$ in general term up to $n = 5$.

$$S_n = \sum_{n=1}^{5}(3n+2) = (3.1+2) + (3.2+2) + (3.3+2) + (3.4+2) + (3.5+2)$$
$$= 5 + 8 + 11 + 14 + 17$$

$$S_5 = 55$$

Number of terms = 5
or Top – bottom+1
(5-1+1)

<u>Example 10:</u>
Determine the value of $\sum_{k=4}^{7} 2k$

<u>Solution:</u>
$$\sum_{k=4}^{7} 2k = 2(4) + 2(5) + 2(6) + 2(7)$$
$$= 8 + 10 + 12 + 14$$

$$S_4 = 44$$

Number of terms = 4
or Top – bottom+1
(7-4+1)

<u>Example 11:</u> Write the following series in sigma notation: $5 + 8 + 11 + 14 + 17$

<u>Solution:</u>
1) First calculate the general term for the series where $a = 5$ and $d = 3$.
   Hence $T_n = a + (n-1)d$
   $T_n = 5 + (n-1)3$
   $T_n = 5 + 3n - 3$
   $T_n = 3n + 2$
2) Write the formula now in sigma notation

$$\sum_{n=\cdots}^{\cdots}(3n+2)$$

Bottom value is the first term which is equal to 5:
$3n + 2 = \mathbf{5}$
$3n = 3$
$n = 1$

Top value is the last term which is equal to 17:
$3n + 2 = \mathbf{17}$
$3n = 15$
$n = 5$

3)	$\sum_{n=1}^{5}(3n+2)$	
CAN YOU?	1) Determine $5+12+19+\cdots+54$ 2) How many terms of the arithmetic series $3+7+11+\cdots$ will add up to 210. 3) Determine the value of the following $\sum_{r=0}^{10}(2r+5)$ 4) Write the following in sigma notation $7+10+13+\cdots+25$	<u>Answers:</u> 1) 236 2) 10 3) 165 4) $\sum_{n=0}^{6}(7+3n)$
ACTIVITIES/ASSESSMENT	see table below	
CONSOLIDATION	see below	

Mind Action Series		Via Afrika		Classroom Mathematics	
Exerise	Page	Exerise	Page	Exerise	Page
2.	5	1.	12	1.3	7
4.	12	3.	22	1.5	14
				1.7	23

CONSOLIDATION

Arithmetic Sequence

$T_1,\ T_2\ ,\ T_3,\ T_4, \ldots\ T_n$

$T_n = a + (n-1)d$ where $\boldsymbol{a} = T_1$ and $\boldsymbol{d} = T_2 - T_1 = T_3 - T_2$

Arithmetic Series

$T_1 + T_2 + T_3 + T_4 + \cdots + T_n = S_n$

$S_n = \frac{n}{2}[2a + (n-1)d]$ or $S_n = \frac{n}{2}[a + l]$

Sigma Notation

$$\sum_{k=1}^{n} \boldsymbol{T_k} = \boldsymbol{S_n}$$