MATHEMATICS
GRADE 9

DATE: ....

TOPIC: Pythagoras' Theorem

CONCEPTS & SKILLS TO BE ACHIEVED: By the end of the lesson learners should know and be able to: • Solve problems using the Theorem of Pythagoras • Use the Theorem of Pythagoras to solve problems involving unknown lengths in geometric figures that contain right-angled triangles	
RESOURCES:	DBE Workbook, Sasol-Inzalo book, Textbooks,
ONLINE RESOURCES	

DAY 1: ACTIVITY 1:

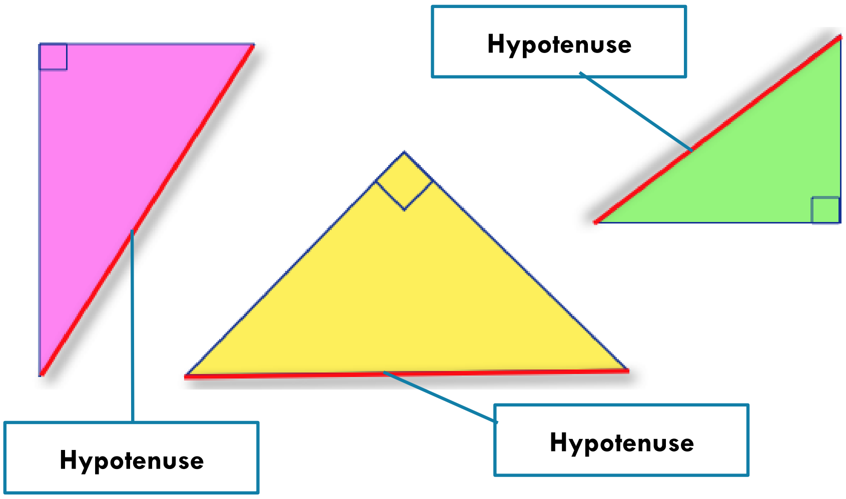
LESSON DEVELOPMENT

DAY 1: Revision:

NOTE TO LEARNER:

1. Pythagoras' Theorem is a rule that applies only to right-angled triangles.

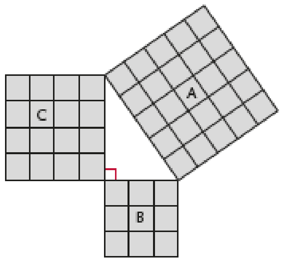
2. A right-angled triangle has one 90° angle. The longest side of the right-angled triangle is called the hypotenuse.



Investigating squares on the sides of right-angled triangles:

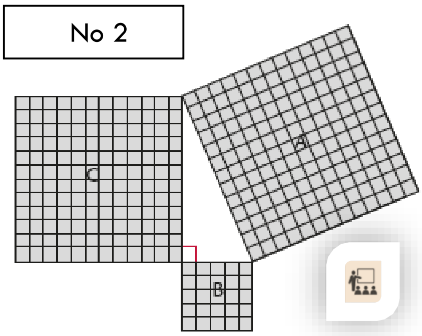
CLASSWORK:

Work through the exercise. Only consult the answers ate the end of the lesson, once you have completed the exercise:

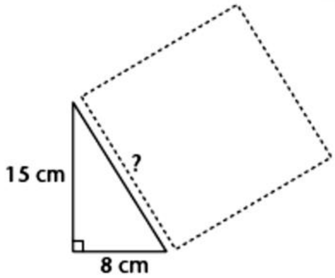


1. The figure shows a right-angled triangle with squares on each of the sides.
(a) Write down the areas (count the small squares) of the following:
Square A
Square B
Square C
(b) Add the area of square B and the area of square C.
(c) What do you notice about the areas?

No 2



2. The figure is similar to the one in question 1. The lengths of the sides of the right-angled triangle are 5 cm and 12 cm.
(a) What is the length of the hypotenuse? Count the squares.
(b) Use the squares to find the following:
Area of A
Area of B
Area of C
Area of B + Area of C
(c) What do you notice about the areas?
Is it similar to your answer in 1(c)?



3. A right-angled triangle has side lengths of 8 cm and 15 cm. Use your findings in the previous questions to answer the following questions:
(a) What is the area of the square drawn along the hypotenuse?
(b) What is the length of the triangle's hypotenuse?

THUS: Pythagoras' Theorem says:

In a right-angled triangle, a square formed on the hypotenuse will have the same area as the sum of the area of the two squares formed on the other sides of the triangle.

We can also reason that the Pythagoras' Theorem applies in two ways:

- **If a triangle is right-angled, the sides will have the following relationship: $(\text{Hypotenuse})^2 = (\text{Side } 1)^2 + (\text{Side } 2)^2$.**
- **If the sides have the relationship: $(\text{Longest side})^2 = (\text{Side } 1)^2 + (\text{Side } 2)^2$, then the triangle is a right-angled triangle.**
- **So, we can test if any triangle is right-angled WITHOUT USING A PROTRACTOR.**

CONSOLIDATION

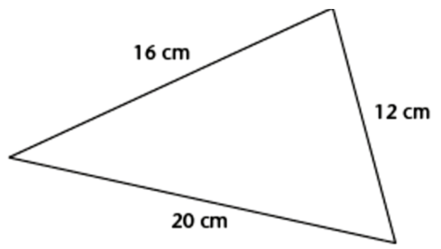
IT IS IMPORTANT TO REMEMBER:

- **If a triangle is right-angled, the sides will have the following relationship: (Hypotenuse)2 = (Side 1)2 + (Side 2)2.**
- **If the sides have the relationship: (Longest side)2 = (Side 1)2 + (Side 2)2, then the triangle is a right-angled triangle.**

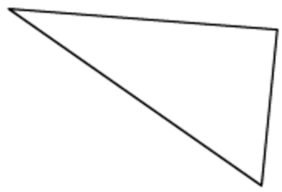
HOMEWORK:

Do the following exercises,applying what you have learnt today. FIRST ATTEMPT TO DO ALL YOUR HOMEWORK BEFORE YOU CHECK YOUR ANSWERS IN THE MEMORANDUM BELOW TODAYS LESSON

1. Is a triangle with sides 12 cm, 16 cm and 20 cm right-angled?



2. This triangle's side lengths are 29 mm, 20 mm and 21 mm.
   (a) Prove that it is a right-angled triangle.
   (b) Copy the triangle and mark the right angle in the diagram.



MEMORANDUM: DAY 1:

<u>CLASSWORK:</u>

1(a) Square A = 25 square units (5 units in length × 5 units in width)
Square B = 9 square units (3 units in length × 3 units in width)
Square C = 16 square units (4 units in length × 4 units in width)

1(b) Square B + Square C
9 square units + 16 square units = 25 square units

1(c) The Area of Square A = Area of square B + Area of square C

2 (a) Length of hypotenuse = 13 squares (13 cm)

2(b) Area of A: 13 cm × 13 cm = 169 cm^2
Area of B: 5 cm × 5 cm = 25 cm^2
Area of C: 12 cm × 12 cm = 144 cm^2
Area of B + Area of C: 144 cm^2 + 25 cm^2 = 169 cm^2

2(c) The sum of the areas of the two squares formed on the sides of the right-angled triangle is equal to the area of the square formed on the hypotenuse.
Yes, it is similar to the answer in 1(c).

3 (a) Area of hypotenuse = Area of 8 cm side + Area of 15 cm side
∴ Area of 8 cm side = 8 cm × 8 cm
= 64 cm^2
∴ Area of 15 cm side = 15 cm × 15 cm
= 225 cm^2
∴ Area of hypotenuse = 64 cm^2 + 225 cm^2
= 289 cm^2

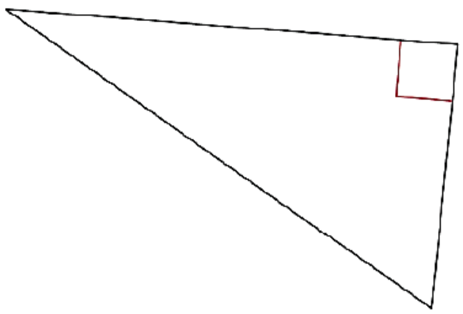
3 (b) Length of hypotenuse = $\sqrt{289}$
= 17 cm

<u>HOMEWORK:</u>

1 If the triangle is a right-angled triangle then:
$(\text{Longest side})^2 = (\text{side } 1)^2 + (\text{side } 2)^2$
LHS $(\text{Longest side})^2$: 20^2 = **400 cm^2**
RHS $(\text{Side } 1)^2 + (\text{side } 2)^2$: $16^2 + 12^2$
= 256 cm^2 + 144 cm^2
= **400 cm^2**
LHS = RHS
∴ Triangle is a right-angled triangle.

2 (a) If the triangle is a right-angled triangle then:
$(\text{Longest side})^2 = (\text{side } 1)^2 + (\text{side } 2)^2$
LHS $(\text{Longest side})^2$: 29^2 = **841 cm^2**
RHS $(\text{Side } 1)^2 + (\text{side } 2)^2$: $20^2 + 21^2$
= 400 cm^2 + 441 cm^2
= **841 cm^2**
LHS = RHS
∴ Triangle is a right-angled triangle.

2 (b)



DAY 2:

LESSON DEVELOPMENT

CLASSWORK:

FINDING MISSING SIDES

IT IS IMPORTANT TO NOTE THAT:

You can use the Pythagoras' Theorem to find the lengths of missing sides if you know that a triangle is right-angled.

Work through the examples that demonstrates finding **the missing hypotenuse:**

1. Calculate the length of the hypotenuse if the lengths of the other two sides are six units and eight units.

ΔABC is right-angled, so: $AC^2 = AB^2 + BC^2$

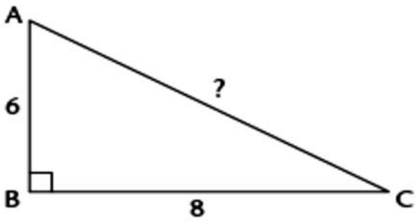
$= (62 + 82)$ units^2

$= 36 + 64$ units^2

$= 100$ units^2

$AC = \sqrt{100}$ units

$= 10$ units



BUT:

Sometimes the square root of a number is not a whole number or a simple fraction. In these cases, you can leave the answer under the square root sign. This form of the number is called a **surd**.

SURD FORM

You pronounce *surd* so that it rhymes with *word*. $\sqrt{5}$ is an example of a number in surd form. $\sqrt{9}$ is not a surd because you can simplify it: $\sqrt{9} = 3$.

Example:

Calculate the length of the hypotenuse

of ΔABC if $\angle B = 90°$, AB = two units and BC = five units.

Leave your answer in surd form, where applicable.

Remember when taking the square root that

length is always positive.

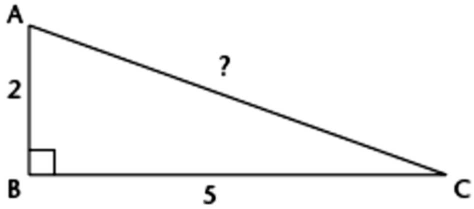
$AC^2 = AB^2 + BC^2$

$= 2^2 + 5^2$ units^2

$= 4 + 25$ units^2

$= 29$ units^2

AC $=\sqrt{29}$ units



CONSOLIDATION

IT IS IMPORTANT TO REMEMBER:

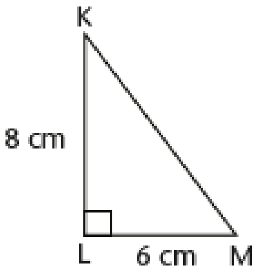
- **If a triangle is right-angled, the sides will have the following relationship: (Hypotenuse)2 = (Side 1)2 + (Side 2)2.**
- **If the sides have the relationship: (Longest side)2 = (Side 1)2 + (Side 2)2, then the triangle is a right-angled triangle.**
- **You can use the Pythagoras' Theorem to find the lengths of missing sides if you know that a triangle is right-angled**

HOMEWORK:

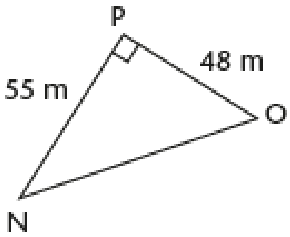
Do the following exercises, applying what you have learnt today. FIRST ATTEMPT TO DO ALL YOUR HOMEWORK BEFORE YOU CHECK YOUR ANSWERS IN THE MEMORANDUM BELOW

1. Find the length of the hypotenuse in each of the triangles shown on the following page. Leave the answers in surd form where applicable.

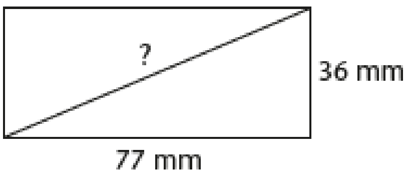
(a)



(b)



2. A rectangle has sides with lengths of 36 mm and 77 mm. Find the length of the rectangle's diagonal.



MEMORANDUM: DAY 2:

HOMEWORK:

1 (a) Because Δ KLM is a right-angled triangle:
$KM^2 = KL^2 + LM^2$
$KM^2 = 8^2 + 6^2$
$KM^2 = 64\ cm^2 + 36\ cm^2$
$KM^2 = 100\ cm^2$
$\therefore KM = \sqrt{100}$
$KM = 10$ cm

1 (b) Because Δ NOP is a right-angled triangle:
$NO^2 = PN^2 + OP^2$
$NO^2 = 55^2 + 48^2$
$NO^2 = 3\ 025\ m^2 + 2\ 304\ m^2$
$NO^2 = 5\ 329\ m^2$
$\therefore NO = \sqrt{5\ 329}$
$NO = 73$ m

2 Because we have a rectangle we know that the angles are 90°
Therefore, we know that the diagonal line will be the hypotenuse
$Diagonal^2 = 77^2 + 36^2$
$Diagonal^2 = 5\ 929\ mm^2 + 1\ 296\ mm^2$
$Diagonal^2 = 7\ 225\ mm^2$
$\therefore Diagonal = \sqrt{7\ 225}$
Length of diagonal = $85\ mm^2$

DAY 3:

LESSON DEVELOPMENT

CLASSWORK:

FINDING MISSING SIDES IN A RIGHT-ANGLED TRIANGLE

IT IS IMPORTANT TO NOTE THAT: You can use the Pythagoras' Theorem to find the lengths of missing sides if you know that a triangle is right-angled.

Work through the examples that demonstrates finding **any missing side in a right-angled triangle**

Example: Find the length of TS in the triangle below.

$US^2 = TU^2 + TS^2$

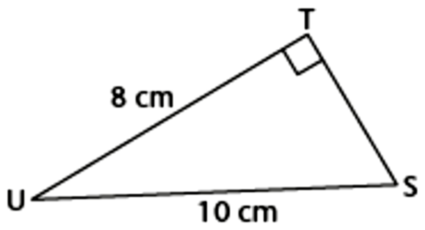
$10^2 = 8^2 + TS^2$

$100 = 64 + TS^2$

$36 = TS^2$

$\sqrt{36} = TS$

$\therefore TS = 6$ cm



CONSOLIDATION

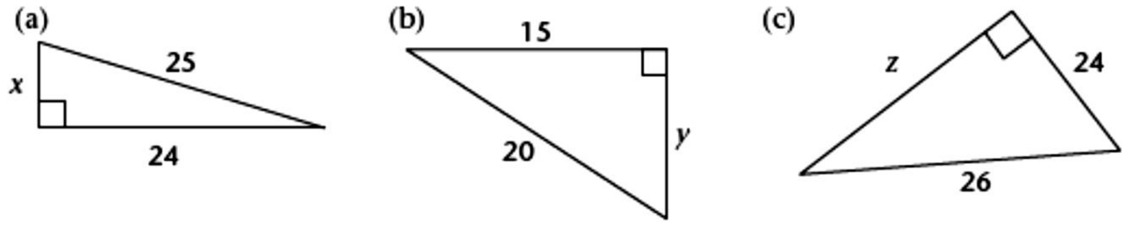
IT IS IMPORTANT TO REMEMBER:

- **If a triangle is right-angled, the sides will have the following relationship: (Hypotenuse)2 = (Side 1)2 + (Side 2)2.**
- **If the sides have the relationship: (Longest side)2 = (Side 1)2 + (Side 2)2, then the triangle is a right-angled triangle.**
- **You can use the Pythagoras' Theorem to find the lengths of any missing side if you know that a triangle is right-angled**

HOMEWORK:

Do the following exercises, applying what you have learnt today. FIRST ATTEMPT TO DO ALL YOUR HOMEWORK BEFORE YOU CHECK YOUR ANSWERS IN THE MEMORANDUM BELOW

1. In the right-angled triangles below, calculate the length of the sides that have not been given. Leave your answers in surd form where applicable.



2. Calculate the length of the third side of each of the following right-angled triangles. First draw a rough sketch of each of the triangles before you do any calculations. Round off your answers to two decimal places.

(a) ΔABC has AB = 12 cm, BC = 18 cm and ∠A = 90°. Calculate AC.

(b) ΔDEF has ∠F = 90°, DE = 58 cm and DF = 41 cm. Calculate EF.

MEMORANDUM: DAY 3:

<u>HOMEWORK:</u>

1 (a)
$25^2 = 24^2 + x^2$
$\therefore 25^2 - 24^2 = x^2$
$625 - 576 = x^2$
$49 = x^2$
$\sqrt{49} = x$
$7 = x$

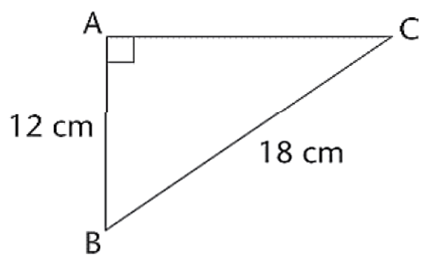
1 (b)
$20^2 = 15^2 + y^2$
$\therefore 20^2 - 15^2 = y^2$
$400 - 225 = y^2$
$175 = y^2$
$\sqrt{175} = y$
$5\sqrt{7} = y$

1(c)
$26^2 = 24^2 + z^2$
$\therefore 26^2 - 24^2 = z^2$
$676 - 576 = z^2$
$100 = z^2$
$\sqrt{100} = z$
$10 = z$

2 (a)
$BC^2 = AC^2 + AB^2$

$18^2 = AC^2 + 12^2$

$\therefore 18^2 - 12^2 = AC^2$
$324 - 144 = AC^2$
$180 \text{ cm}^2 = AC^2$
$\sqrt{180} = AC$
$13{,}42 \text{ cm} = AC$



2 (b) $DE^2 = DF^2 + EF^2$

$58^2 = 41^2 + EF^2$

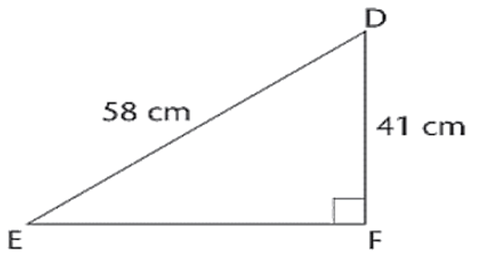
$\therefore 58^2 - 41^2 = EF^2$

$3\,364 - 1\,681 = EF^2$

$1\,683 \text{ cm}^2 = EF^2$

$\sqrt{1\,683} = EF$

$41{,}02 \text{ cm} = EF$



DAY 4:

LESSON DEVELOPMENT

CLASSWORK:

REVISE THE CHARACTERISTIC OF THE Pythagoras' Theorem

You will need to apply the Pythagoras' Theorem in solving the problems

HOMEWORK:

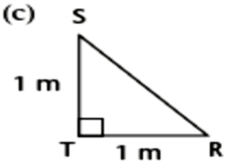
Do the following exercises, applying what you have learnt today. FIRST ATTEMPT TO DO ALL YOUR HOMEWORK BEFORE YOU CHECK YOUR ANSWERS IN THE MEMORANDUM BELOW

1. Determine whether the following side lengths would form right-angled triangles. All values are in the same units.

(a) 6, 8 and 10 b) 8, 15 and 17 (c) 16, 21 and 25

2. Find the length of the hypotenuse in the triangle. Leave the answers in surd form where applicable.

(c)



3. Calculate the length of the third side of each of the following right-angled triangles. First draw a rough sketch of each of the triangles before you do any calculations.

Round off your answers to two decimal places.

(a) ΔJKL has $\angle K = 90°$, $JK = 119$ m and $KL = 167$ m. Calculate JL.

(b) ΔPQR has $PQ = 2$ cm, $QR = 8$ cm and $\angle Q = 90°$. Calculate PR

MEMORANDUM: DAY 4:

<u>HOMEWORK:</u>

1 (a) If the triangle is a right-angled triangle then:

(Longest side)^2 = (side 1)^2 + (side 2)^2

LHS (Longest side)^2: 10^2 = **100 units^2**

RHS (Side 1)^2 + (side 2)^2: $6^2 + 8^2$

= 36 units^2 + 64 units^2

= **100 units^2**

LHS = RHS

∴ Those three side lengths will form a right-angled triangle.

1 (b) If the triangle is a right-angled triangle then:
$(\text{Longest side})^2 = (\text{side } 1)^2 + (\text{side } 2)^2$
LHS $(\text{Longest side})^2$: $17^2 =$ **289 units²**
RHS $(\text{Side } 1)^2 + (\text{side } 2)^2$: $8^2 + 15^2$
$= 64 \text{ units}^2 + 225 \text{ units}^2$
$=$ **289 units²**
LHS = RHS
$\therefore$ Those three side lengths will form a right-angled triangle.

1 (c) If the triangle is a right-angled triangle then:
$(\text{Longest side})^2 = (\text{side } 1)^2 + (\text{side } 2)^2$
LHS $(\text{Longest side})^2$: $25^2 =$ **625 units²**
RHS $(\text{Side } 1)^2 + (\text{side } 2)^2$: $16^2 + 21^2$
$= 256 \text{ units}^2 + 441 \text{ units}^2$
$=$ **697 units²**
LHS $\neq$ RHS
$\therefore$ Those three side lengths will not form a right-angled triangle.

2(a) $SR^2 = TR^2 + ST^2$

$SR^2 = 1^2 + 1^2$

$SR^2 = 1 + 1$

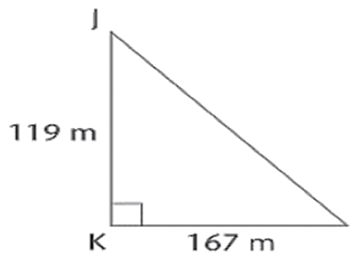
$SR^2 = 2 \text{ m}^2$

$SR = \sqrt{2} \text{ m}^2$

3 (a) $JL^2 = KL^2 + JK^2$

$JL^2 = 167^2 + 119^2$

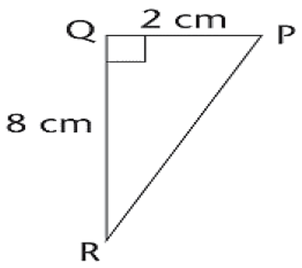
$JL^2 = 27\,889 + 14\,161$
$JL^2 = 42\,050 \text{ m}^2$
$JL = \sqrt{42050} \text{ m}$
$JL = 205{,}06 \text{ m}$



3 (b) $PR^2 = QP^2 + QR^2$

$PR^2 = 2^2 + 8^2$

$PR^2 = 4 + 64$
$PR^2 = 68 \text{ cm}^2$
$PR = \sqrt{68} \text{ cm}$
$PR = 8{,}25 \text{ cm}$



DAY 5:

LESSON DEVELOPMENT

CLASSWORK:

REVISE THE CHARACTERISTIC OF THE Pythagoras' Theorem

You will need to apply the Pythagoras' Theorem in solving the problems

HOMEWORK:

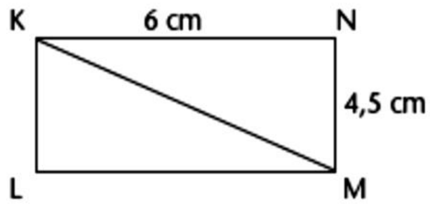
Do the following exercises, applying what you have learnt today. FIRST ATTEMPT TO DO ALL YOUR HOMEWORK BEFORE YOU CHECK YOUR ANSWERS IN THE MEMORANDUM BELOW

1. a) A ladder with a length of 5 m is placed at an angle against a wall. The bottom of the ladder is 1 m away from the wall. How far up the wall will the ladder reach? Round off to two decimal places.

   (b) If the ladder reaches a height of 4,5 m against the wall, how far away from the wall was it placed? Round off to two decimal places

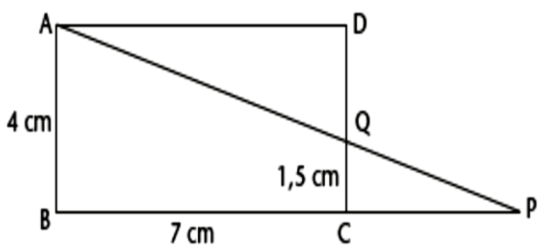
2. a) Calculate the area of rectangle KLMN.

   (b) Calculate the perimeter of ΔKLM.



3. ABCD is a rectangle with AB = 4 cm, BC = 7 cm and CQ = 1,5 cm. Round off your answers to two decimal places if the answers are not whole numbers.

(a) What is the length of QD?
(b) If CP = 4,2 cm, calculate the length of PQ.
(c) Calculate the length of AQ and the area of ΔAQD

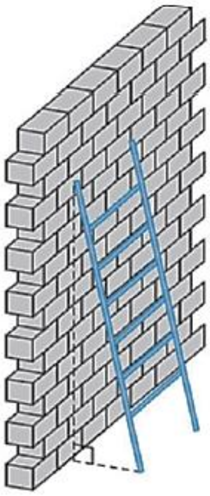


MEMORANDUM: DAY 5:

HOMEWORK:

1(a) You will see from the diagram to the right that we have a right-angled triangle between the ladder and the wall.
Let the height against the wall be ***h***, Length of the ladder ***L*** and the distance from the wall ***d***.

$L^2 = h^2 + d^2$
$5^2 = h^2 + 1^2$
$5^2 - 1^2 = h^2$
$25 - 1 = h^2$
$24 \text{ m}^2 = h^2$
$\therefore h = \sqrt{24}$ m
$h = 4{,}90$ m



1 (b) $L^2 = h^2 + d^2$
$5^2 = 4{,}5^2 + d^2$
$25 - 20{,}25 = d^2$
$4{,}75 \text{ m}^2 = d^2$
$\therefore d = \sqrt{4{,}75}$ m
d = 2,18 m

2 (a) Area = length × breadth
Area = 6 cm × 4,5 cm
Area = 27 cm^2

2 (b) $KM^2 = LM^2 + KL^2$ **Because we have a rectangle KLMN, LM = KN & KL = NM**

$KM^2 = 6^2 + 4{,}5^2$

$KM^2 = 36 + 20{,}25$
$KM^2 = 56{,}25 \text{ cm}^2$
$KM = \sqrt{56{,}25}$ cm
KM =7,5 cm

$\therefore$ Perimeter KLM = KM + LM + KL
Perimeter = 7,5 cm + 6 cm + 4,5 cm
Perimeter = 18 cm

3 (a) QD = DC – QC **Because it is a rectangle DC = AB**
QD = 4 cm – 1,5 cm
QD = 2,5 cm

3 (b) $QP^2 = CP^2 + QC^2$

$QP^2 = 4{,}2^2 + 1{,}5^2$

$QP^2 = 17{,}64 + 2{,}25$
$QP^2 = 19{,}89 \text{ cm}^2$
$QP = \sqrt{19{,}89}$ cm
QP =4,46 cm

3 (c) $AQ^2 = AD^2 + DQ^2$

$AQ^2 = 7^2 + 2{,}5^2$

$AQ^2 = 49 + 6{,}25$
$AQ^2 = 55{,}25 \text{ cm}^2$
$AQ = \sqrt{55{,}25} \text{ cm}$
$AQ = 7{,}43 \text{ cm}$

$\therefore$ Area of Δ AQD = ½ base × ⊥ height
Area = ½ × 7 × 2,5
Area = $8{,}75 \text{ cm}^2$